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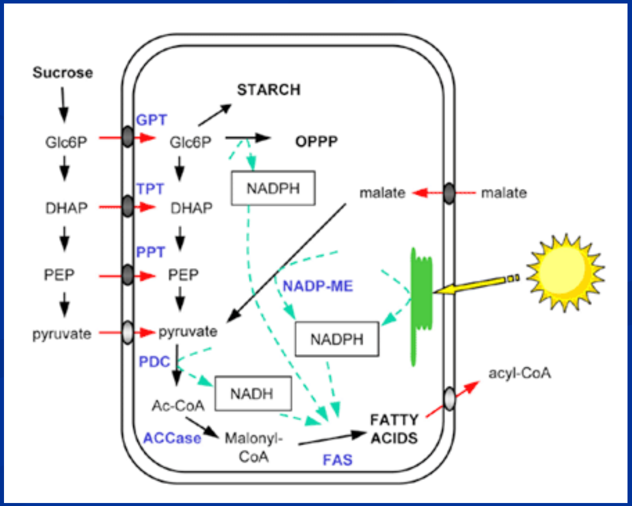
Universidade Federal Fluminense

Redes complexas

Marcio Argollo de Menezes

Universidade Federal Fluminense

Niterói, Rio de Janeiro



Redes: um paradigma de sistemas interagentes

Redes

- Físicas: Estruturas estáticas/dinâmicas
- Relacionais: Sociais, econômicas

Cristalografia, teoria de grupos, estrutura eletrônica

↓

Relevância da estrutura de uma rede em propriedades estruturais e de transporte

Redes: um paradigma de sistemas interagentes

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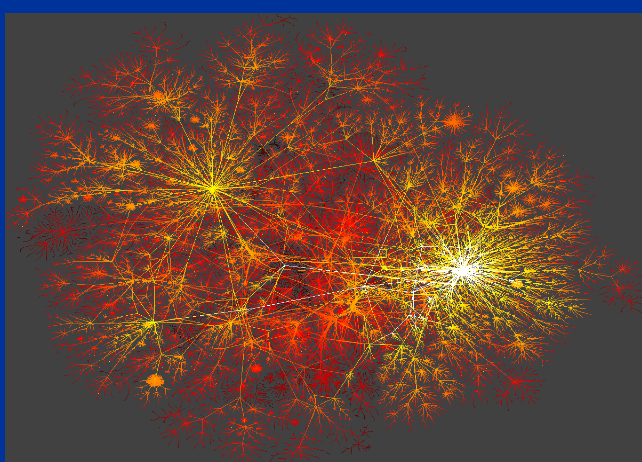
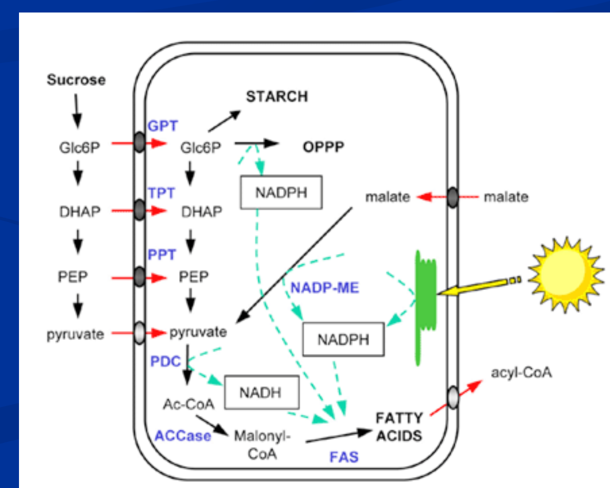
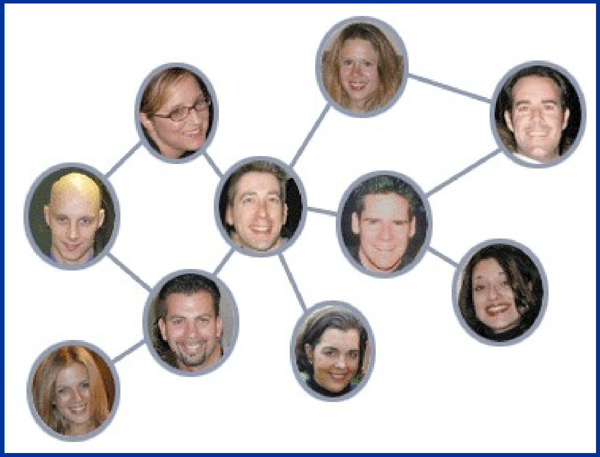




A experiência de Milgram

Stanley Milgram "The Small World Problem”, Psychology Today 2: 60-67 (1967).
http://en.wikipedia.org/wiki/Small_world_phenomenon

- Remetentes aleatoriamente escolhidos na costa leste dos EUA
- Destinatário na costa oeste
- Se remetente conhece o destinatário pessoalmente, entrega a carta
- Se não conhecer, entrega para amigo que possa conhecer o destinatário



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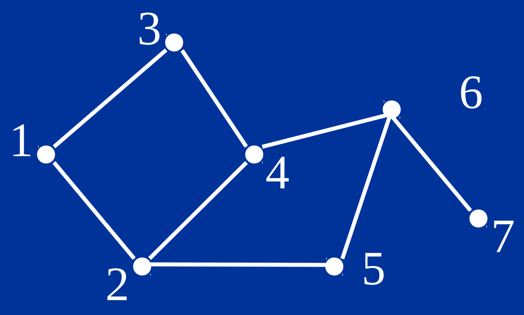

$l_{15} = 2$ [1→2→5]

$l_{17} = 4$ [1→3→4→6 → 7]

… $< \boldsymbol{l} > =$ **??**

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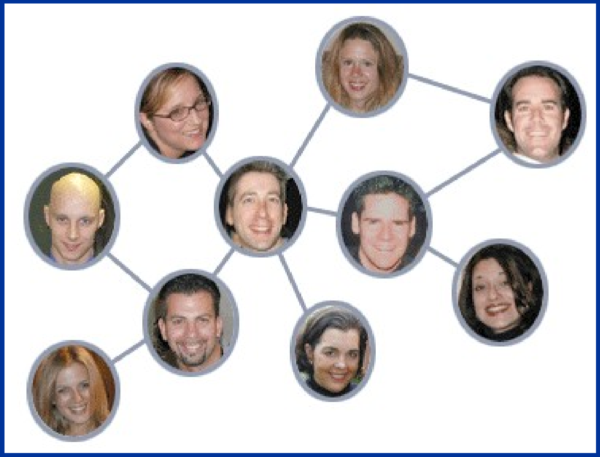

“Six degrees of freedom”

$N \sim 10^6$

$\langle l \rangle \sim 6$

	network	type	n	m	z	ℓ
social	film actors	undirected	449 913	25 516 482	113.43	3.48
	company directors	undirected	7 673	55 392	14.44	4.60
	math coauthorship	undirected	253 339	496 489	3.92	7.57
	physics coauthorship	undirected	52 909	245 300	9.27	6.19
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92
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information	WWW nd.edu	directed	269 504	1 497 135	5.55	11.27
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18
	citation network	directed	783 339	6 716 198	8.57	
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87
	word co-occurrence	undirected	460 902	17 000 000	70.13	
technological	Internet	undirected	10 697	31 992	5.98	3.31
	power grid	undirected	4 941	6 594	2.67	18.99
	train routes	undirected	587	19 603	66.79	2.16
	software packages	directed	1 439	1 723	1.20	2.42
	software classes	directed	1 377	2 213	1.61	1.51
	electronic circuits	undirected	24 097	53 248	4.34	11.05
	peer-to-peer network	undirected	880	1 296	1.47	4.28
biological	metabolic network	undirected	765	3 686	9.64	2.56
	protein interactions	undirected	2 115	2 240	2.12	6.80
	marine food web	directed	135	598	4.43	2.05
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	neural network	directed	307	2 359	7.68	3.97

Grafos Aleatórios de Erdös-Rényi

P. Erdos and A. Renyi. On random graphs. Publ. Math. Debrecen, 6:290--297, 1959.

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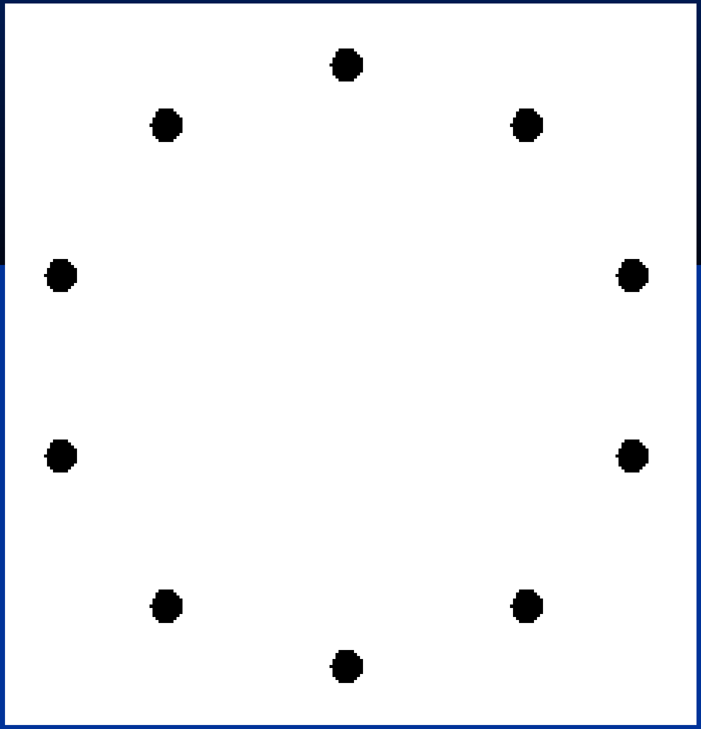

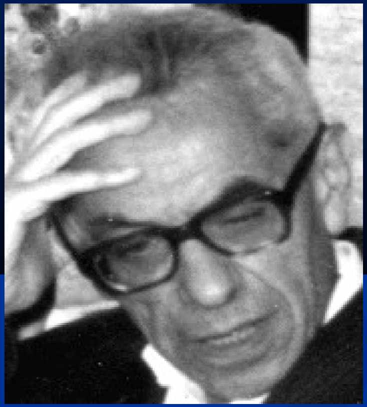

Pál Erdös

(1913-1996)

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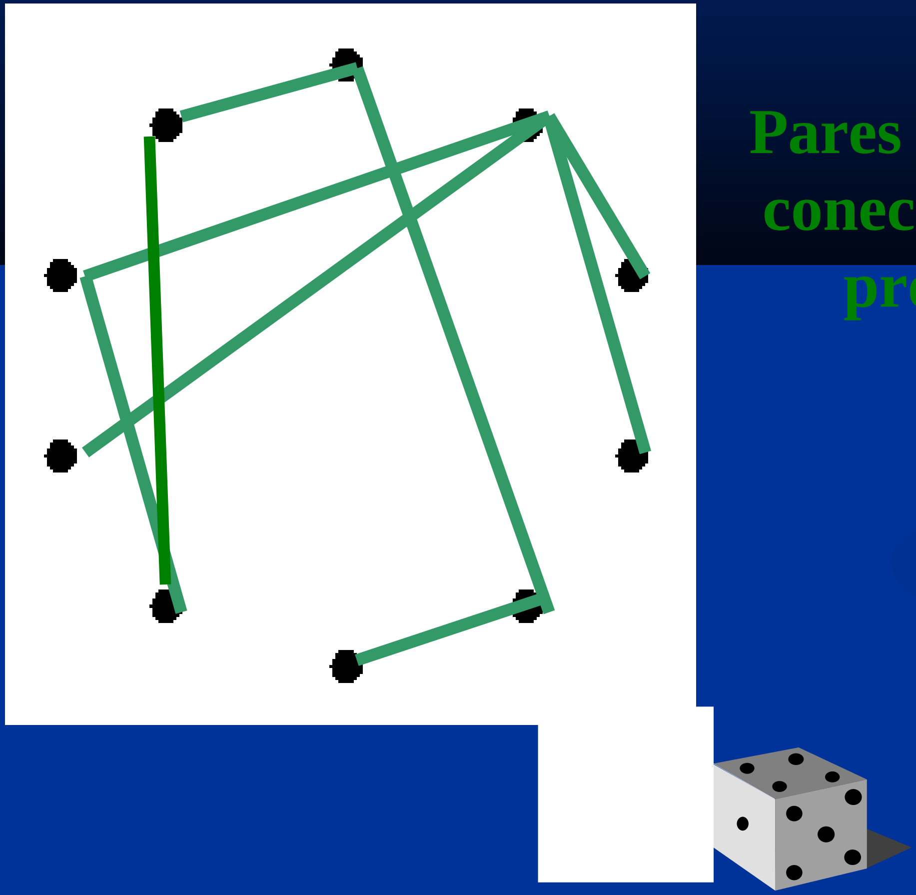
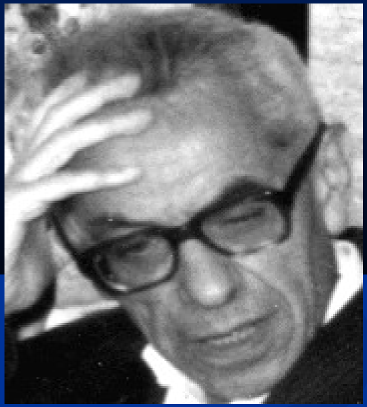


Pares de sítios conectados c/ prob. p



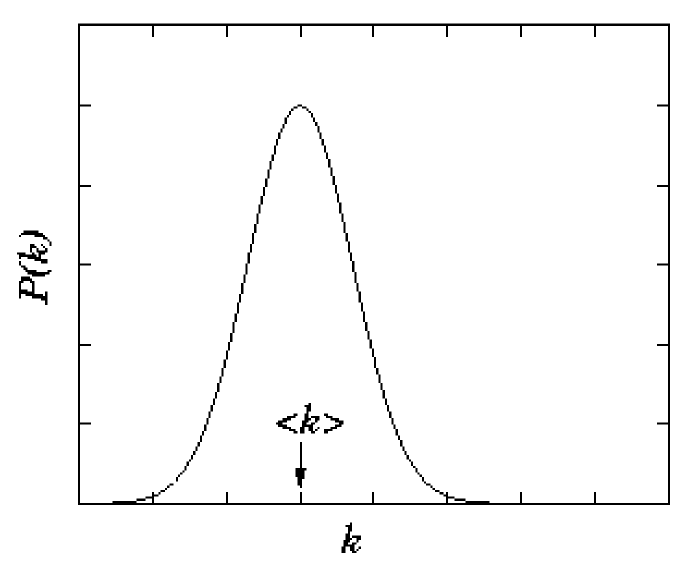
Pál Erdös

(1913-1996)

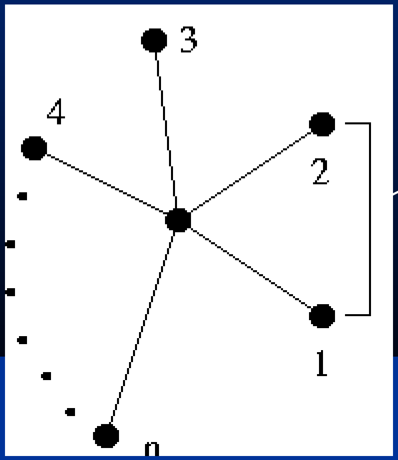
- The $G_{n,p}$ model
  - n : the number of vertices
  - $0 \le p \le 1$
  - for each pair (i,j), generate the edge (i,j) independently with probability p

$$p(k) = \frac{z^k}{k!} e^{-z}, \quad z = \langle k \rangle = Np$$

Poisson distribution



E o amigo do amigo?



Probability to be connected = C

$$C = \frac{\text{\# of links between } 1,2,\ldots n \text{ neighbors}}{n(n-1)/2}$$

Na rede aleatória

$C \sim p = \langle k \rangle / N$

Loops são improváveis! Localmente temos árvores…

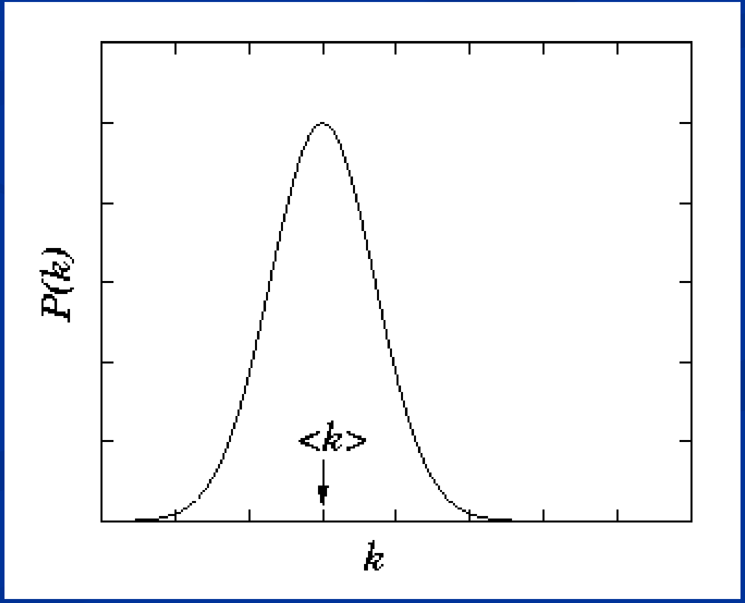
- The $G_{n,p}$ model
  - n : the number of vertices
  - $0 \le p \le 1$
  - for each pair (i,j), generate the edge (i,j) independently with probability p

$$p(k) = \frac{z^k}{k!} e^{-z}, \quad z = \langle k \rangle = Np$$

$$C \sim \frac{\langle k \rangle}{N} \rightarrow 0$$

$$\langle \ell \rangle \sim \frac{\log(N)}{\log(z)}$$

Poisson distribution



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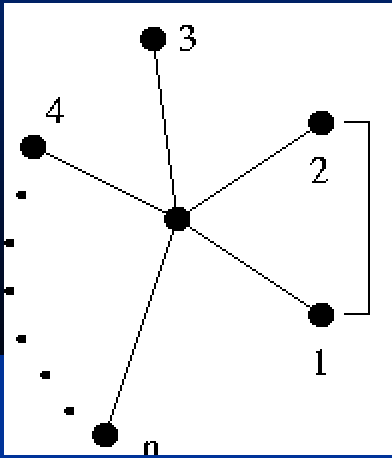


Probability to be connected

$$C = \frac{\text{\# of links between 1,2,…n neighbors}}{n(n-1)/2}$$

Na rede aleatória

C ~ p = <k>/N

Em redes empíricas

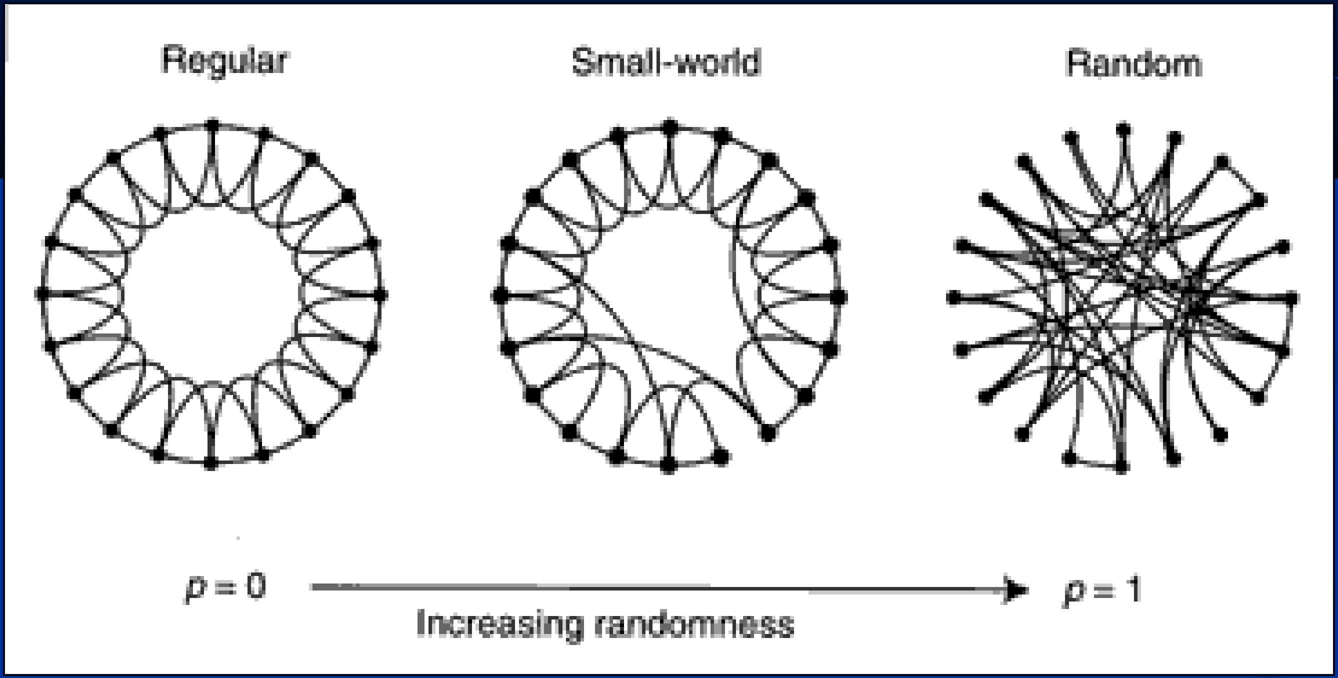
C >> 1/N

Table 1: Clustering coefficients, C, for a number of different networks; n is the number of node, z is the mean degree. Taken from [146].

Network	n	z	C measured	C for random graph
Internet [153]	6,374	3.8	0.24	0.00060
World Wide Web (sites) [2]	153,127	35.2	0.11	0.00023
power grid [192]	4,941	2.7	0.080	0.00054
biology collaborations [140]	1,520,251	15.5	0.081	0.000010
mathematics collaborations [141]	253,339	3.9	0.15	0.000015
film actor collaborations [149]	449,913	113.4	0.20	0.00025
company directors [149]	7,673	14.4	0.59	0.0019
word co-occurrence [90]	460,902	70.1	0.44	0.00015
neural network [192]	282	14.0	0.28	0.049
metabolic network [69]	315	28.3	0.59	0.090
food web [138]	134	8.7	0.22	0.065

Redes Small World

Duncan J. Watts & Steven H. Strogatz,
"Collective dynamics of 'small-world' networks". Nature 393: 440-442 (1998).



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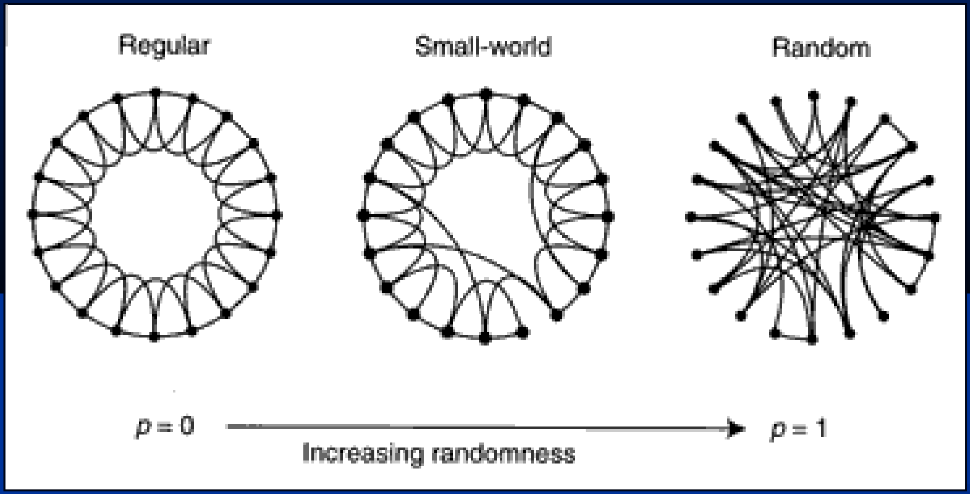

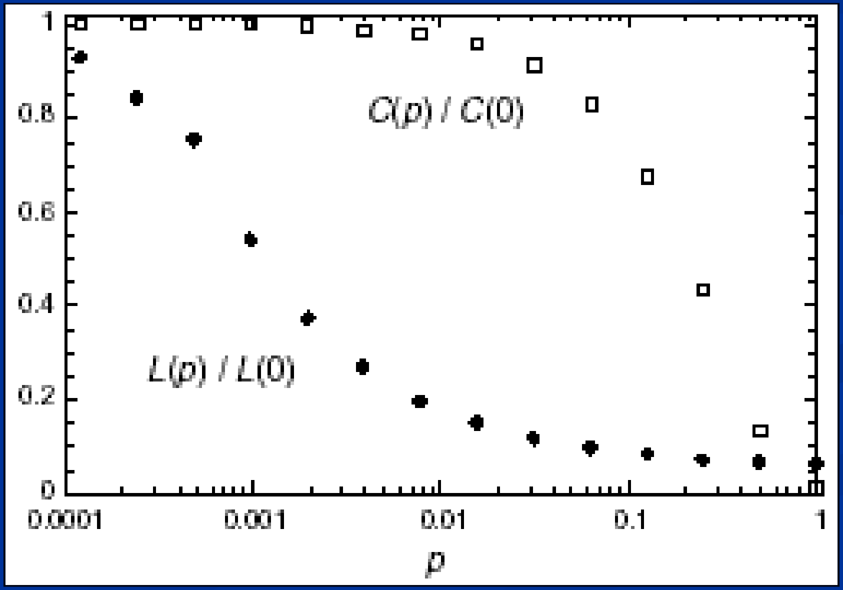

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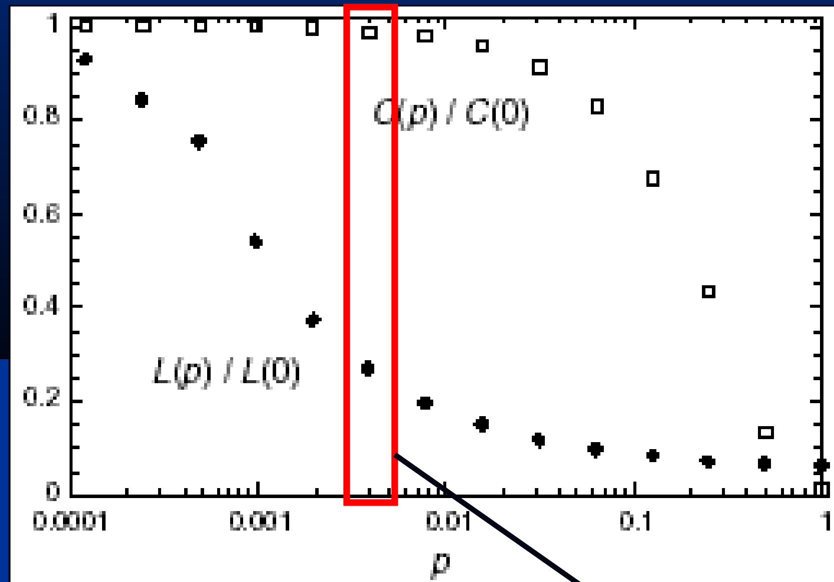
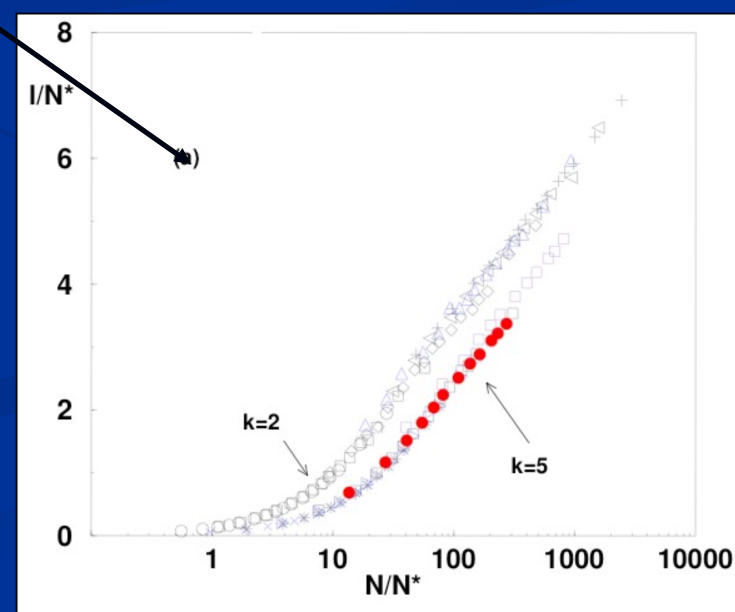
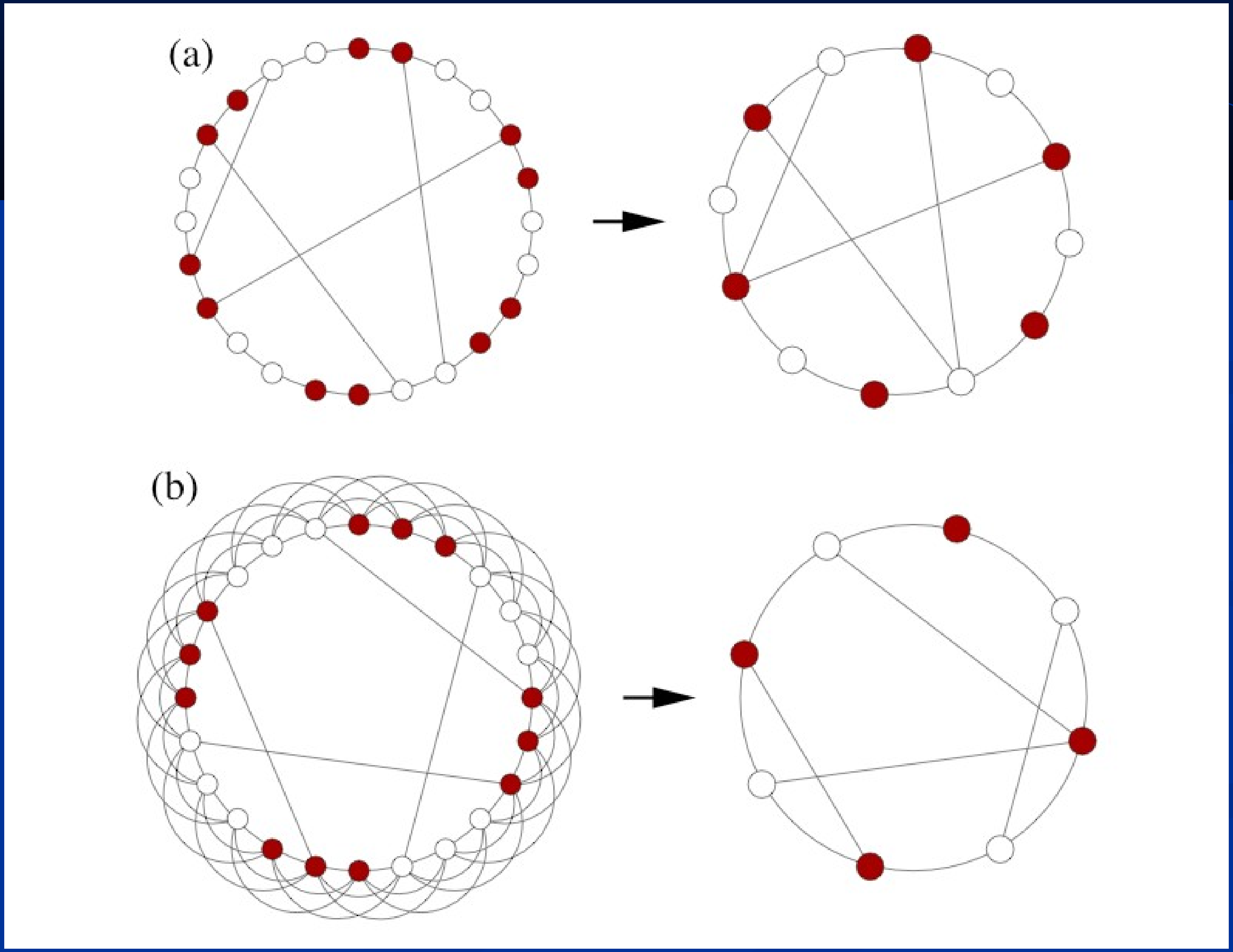




Grupo de renormalização e redes small-world:

``First-order phase transition on small-world networks",
M. Argollo de Menezes, C.F. Moukarzel and TJP Penna, Europhys. Lett. 50, 5 (2000
)



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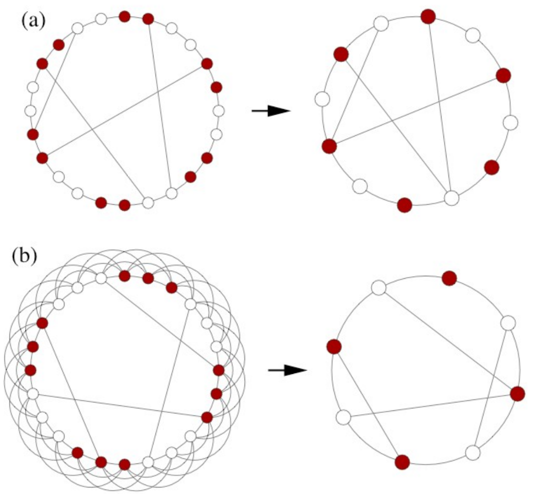
“No limite termodinâmico (N→∞) redes SW têm mesma propriedades topológicas de redes aleatórias (Erdös-Rényi)”



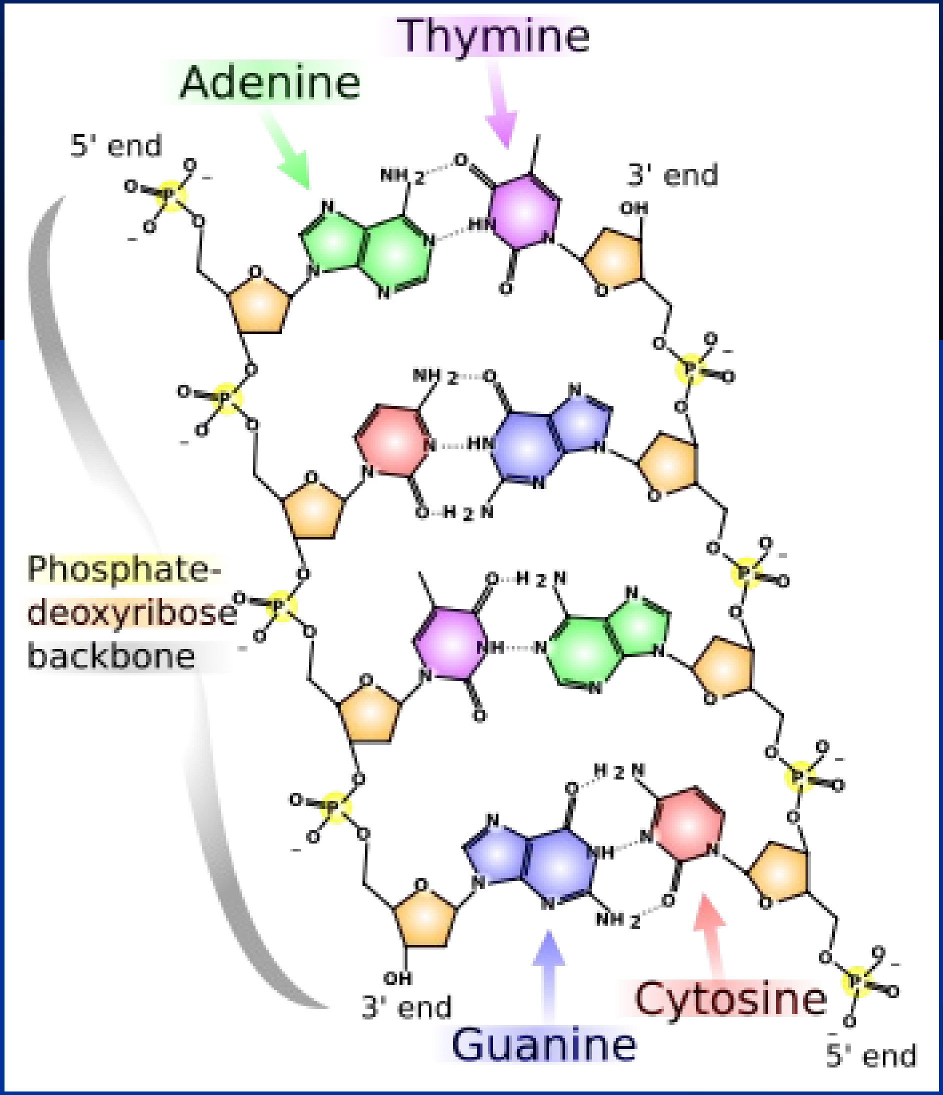
$$\log(1-p^{(N)}) = b^{(2d)} \log(1-p^{(bN)})$$

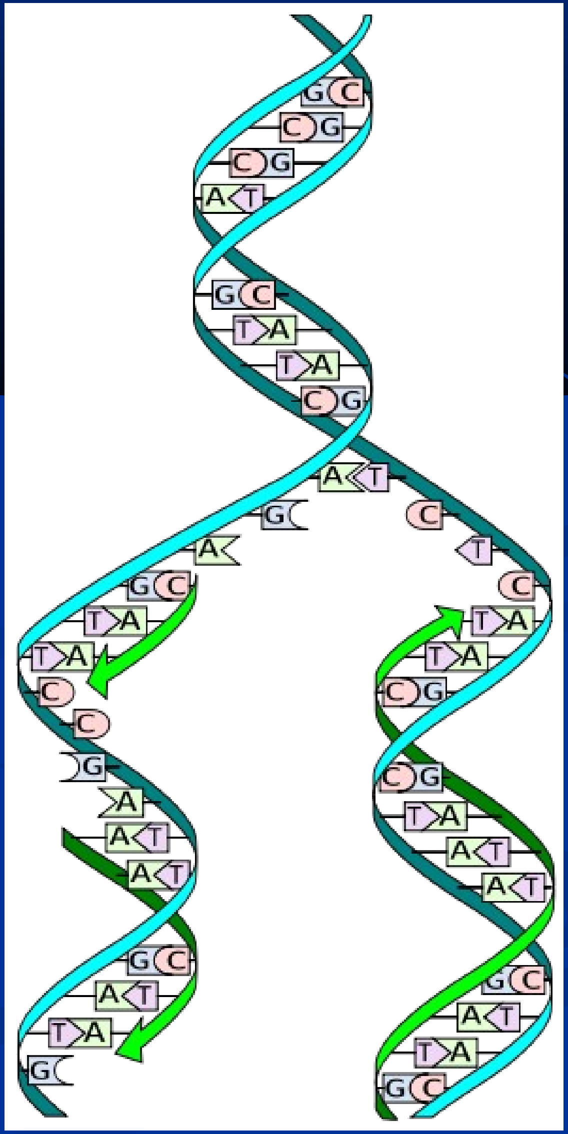
$$\ell(N,p) = L f\left(\frac{L}{L^*}\right) \to \begin{cases} A, \; L \ll L^* \\ \frac{\log(L/L^*)}{L/L^*}, \; L \gg L^* \end{cases}$$

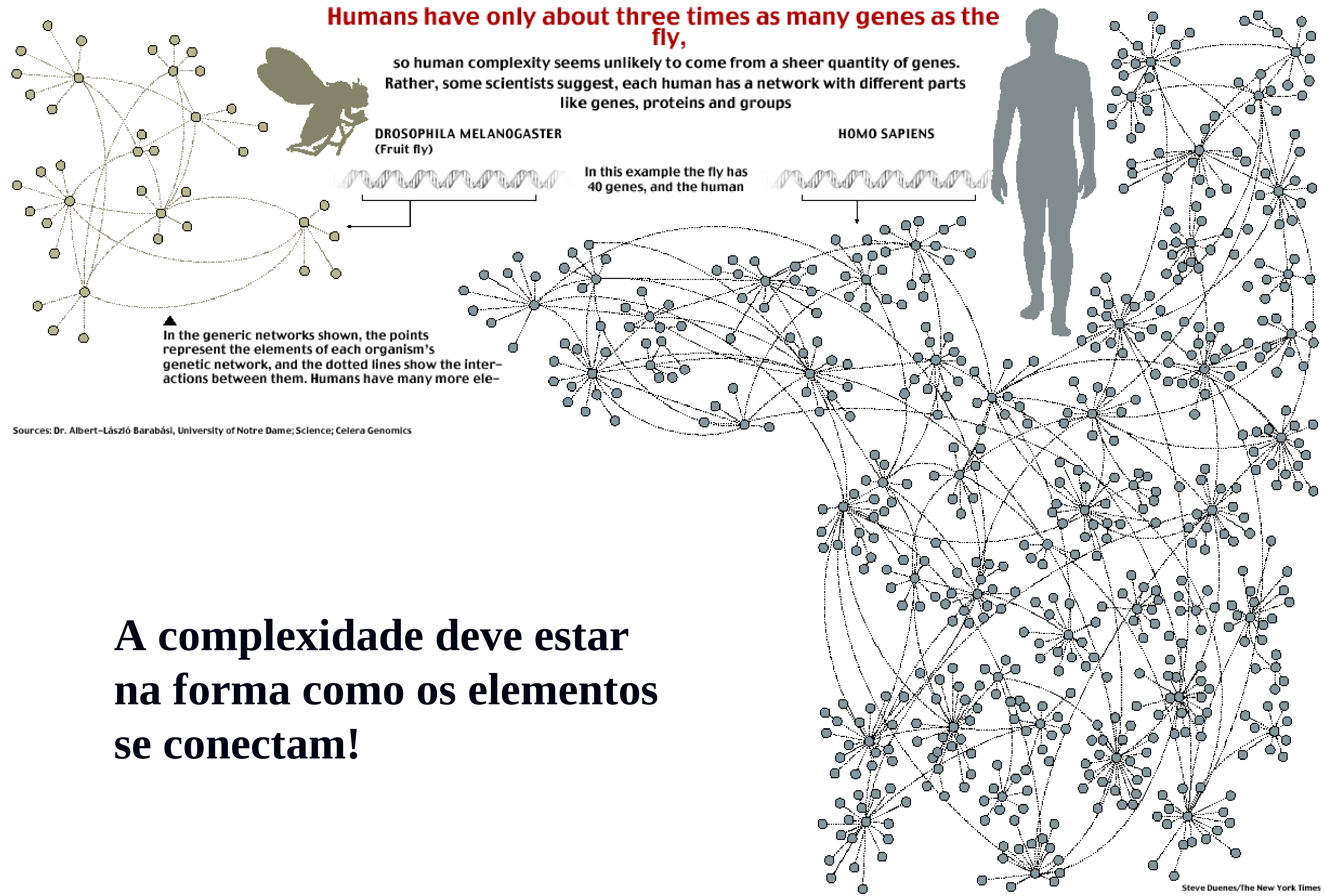
$p^*=0$ $p^*=1$

$$L^* \sim p^\tau, \; \tau = 1/d$$

O DNA: a memória da receita da vida







A complexidade deve estar na forma como os elementos se conectam!

A Internet

- 1957 - União Soviética lança com sucesso o Sputnik
- 1962 - DoD cria sistema de informação resistente a desastres (nucleares)

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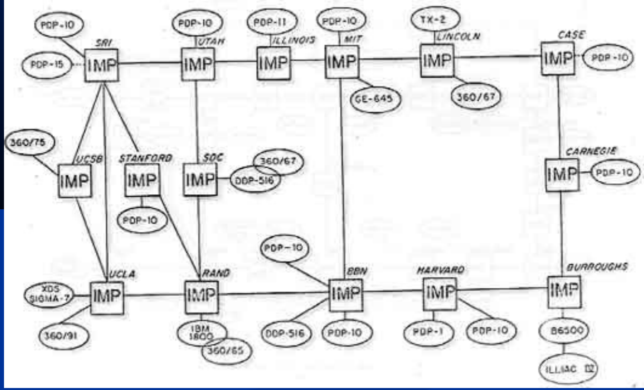
- Militares transferem sistema para universidades (ARPANET)



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- NSFNET - Backbone + sub-redes (MCI, Sprint)

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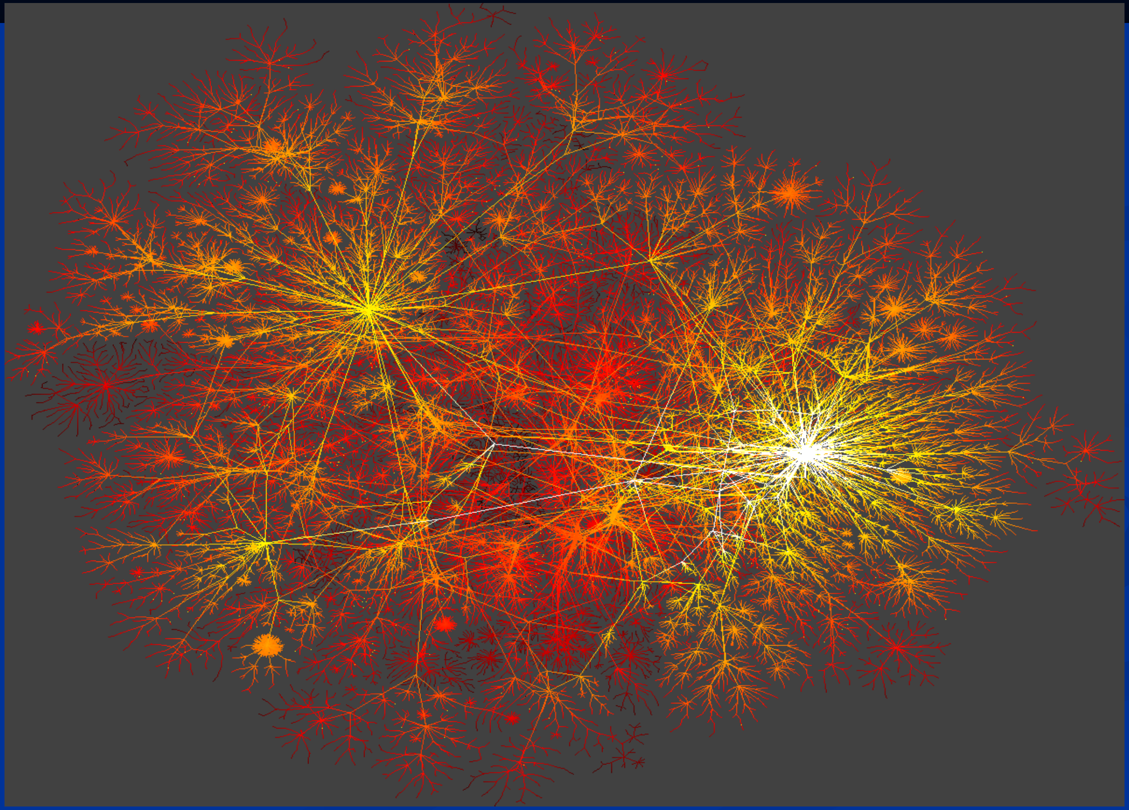
- Militares transferem sistemas para universidades (ARPANET)
- NSFNET - Backbone + sub-redes (MCI, Sprint)
- Aumento exponencial no número de sub-redes



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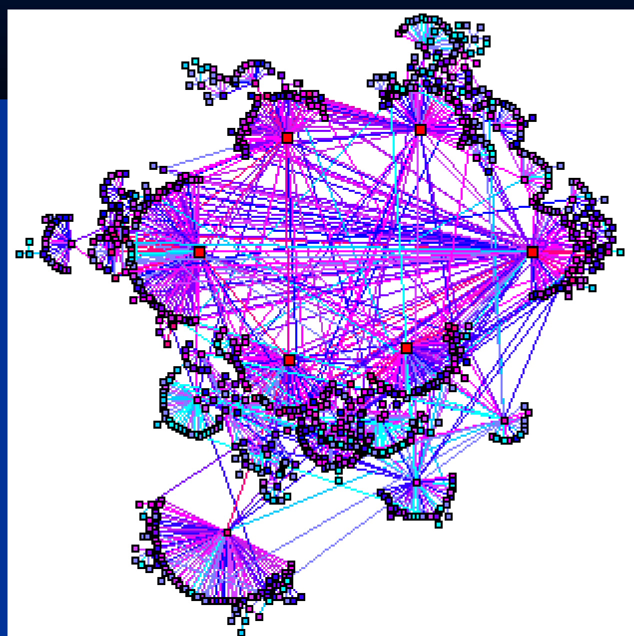
World Wide Web

Nodes: WWW documents
Links: URL links

800 million documents
(S. Lawrence, 1999)



ROBOT: **collects all URL's found in a document and follows them recursively**

R. Albert, H. Jeong, A-L Barabasi, Nature, **401** 130 (1999)

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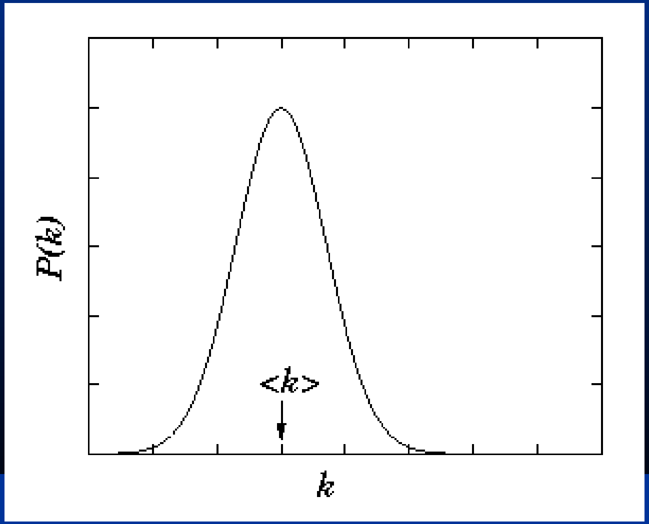
What did we expect?



$\langle k \rangle \sim 6$

$P(k=500) \sim 10^{-99}$

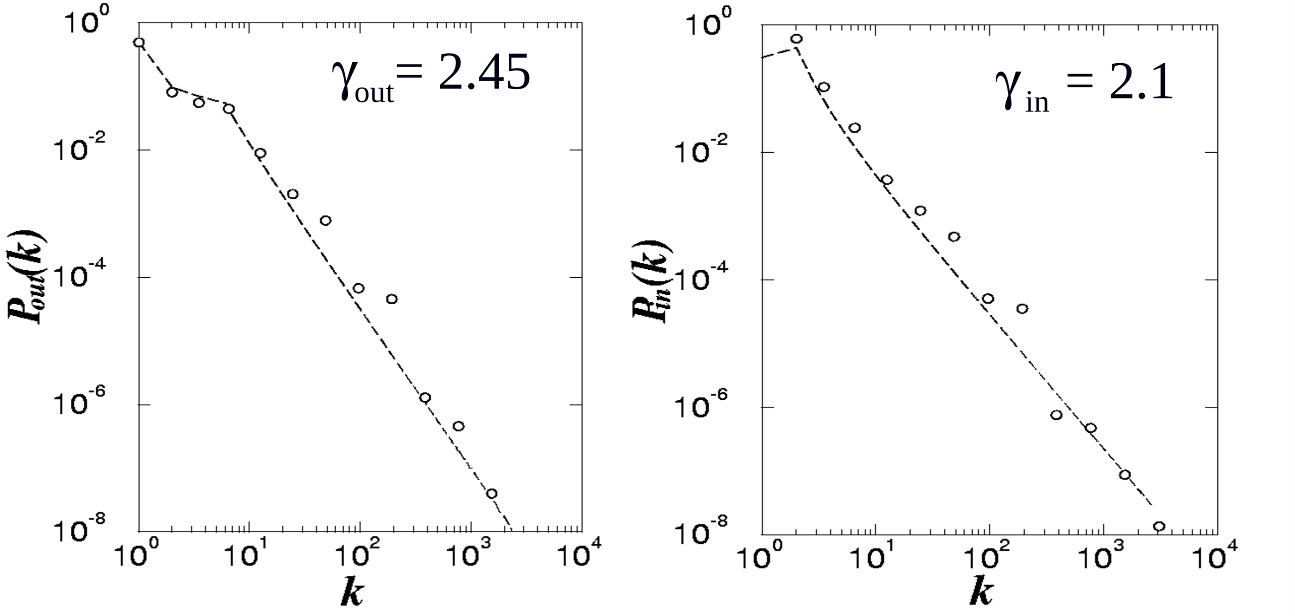
$N_{WWW} \sim 10^{9}$

$\Rightarrow N(k=500) \sim 10^{-90}$

We find:

$$P_{out}(k) \sim k^{-\gamma_{out}}$$

$$P_{in}(k) \sim k^{-\gamma_{in}}$$



$P(k=500) \sim 10^{-6}$

$N_{WWW} \sim 10^{9}$

$\Rightarrow N(k=500) \sim 10^{3}$

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	network	type	n	m	z	ℓ	α	$C^{(1)}$
social	film actors	undirected	449 913	25 516 482	113.43	3.48	2.3	0.20
	company directors	undirected	7 673	55 392	14.44	4.60	–	0.59
	math coauthorship	undirected	253 339	496 489	3.92	7.57	–	0.15
	physics coauthorship	undirected	52 909	245 300	9.27	6.19	–	0.45
	biology coauthorship	undirected	1 520 251	11 803 064	15.53	4.92	–	0.088
	telephone call graph	undirected	47 000 000	80 000 000	3.16		2.1	
	email messages	directed	59 912	86 300	1.44	4.95	1.5/2.0	
	email address books	directed	16 881	57 029	3.38	5.22	–	0.17
	student relationships	undirected	573	477	1.66	16.01	–	0.005
	sexual contacts	undirected	2 810				3.2	
information	WWW `nd.edu`	directed	269 504	1 497 135	5.55	11.27	2.1/2.4	0.11
	WWW Altavista	directed	203 549 046	2 130 000 000	10.46	16.18	2.1/2.7	
	citation network	directed	783 339	6 716 198	8.57		3.0/–	
	Roget's Thesaurus	directed	1 022	5 103	4.99	4.87	–	0.13
	word co-occurrence	undirected	460 902	17 000 000	70.13		2.7	
technological	Internet	undirected	10 697	31 992	5.98	3.31	2.5	0.035
	power grid	undirected	4 941	6 594	2.67	18.99	–	0.10
	train routes	undirected	587	19 603	66.79	2.16	–	
	software packages	directed	1 439	1 723	1.20	2.42	1.6/1.4	0.070
	software classes	directed	1 377	2 213	1.61	1.51	–	0.033
	electronic circuits	undirected	24 097	53 248	4.34	11.05	3.0	0.010
	peer-to-peer network	undirected	880	1 296	1.47	4.28	2.1	0.012
biological	metabolic network	undirected	765	3 686	9.64	2.56	2.2	0.090
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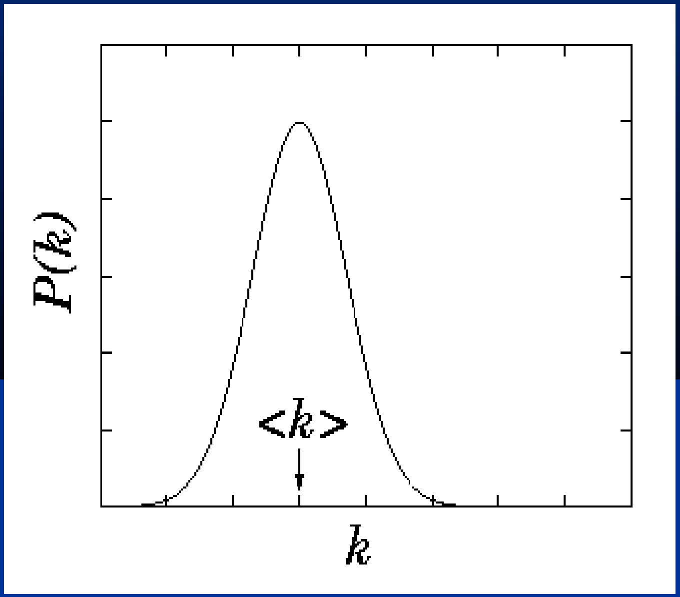
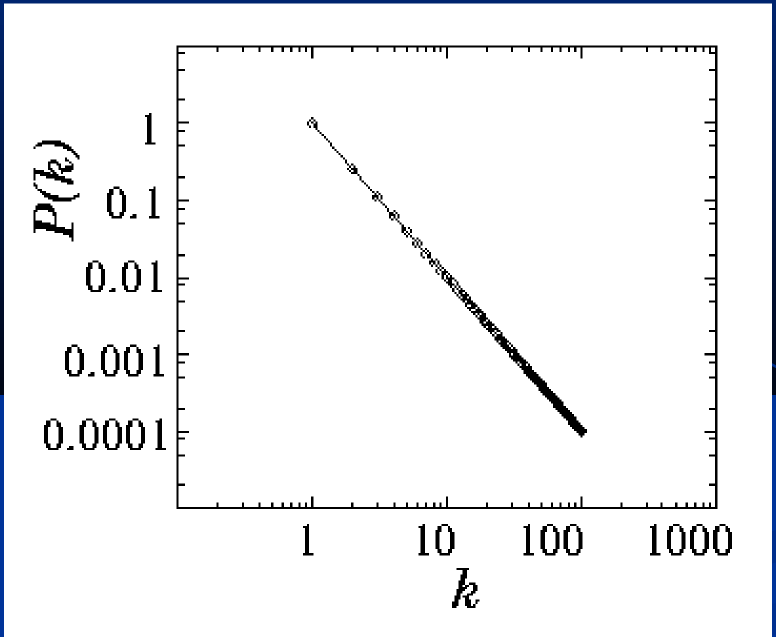
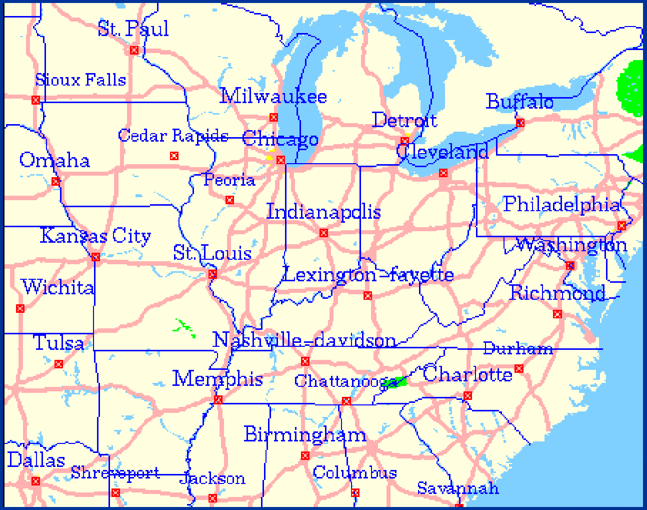
O que isso significa?







Exponential

Scale-free

$$q_k = \frac{(k+1)p_{k+1}}{\sum_k kp_k}$$

de vizinhos de um dos vizinhos de um sítio i= $\sum_k kq_k$

$$z_2 = \langle k \rangle \sum_k kq_k = \langle k \rangle \left(\frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} \right)$$

$$z_m = \left(\frac{z_2}{z_1} \right)^{m-1} z_1$$

$$G_0(x) = \sum_k p_k x^x$$

$$G_1(x) = \sum_k q_k x^x$$

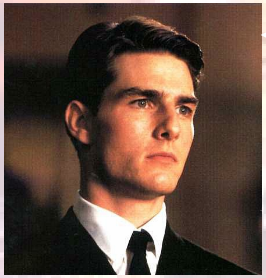
$$G_0(1) = 1,\ G_0'(1) = \langle k \rangle$$

$$G_1'(1) = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle}$$

$$[G_0(x)]^n = \sum_{i_1, i_2, \ldots, i_n} p_{k_1} p_{k_2} \cdots p_{k_n} x^{k_1+k_2+\ldots+k_n}$$

ACTOR CONNECTIVITIES

Nodes: actors
Links: cast jointly

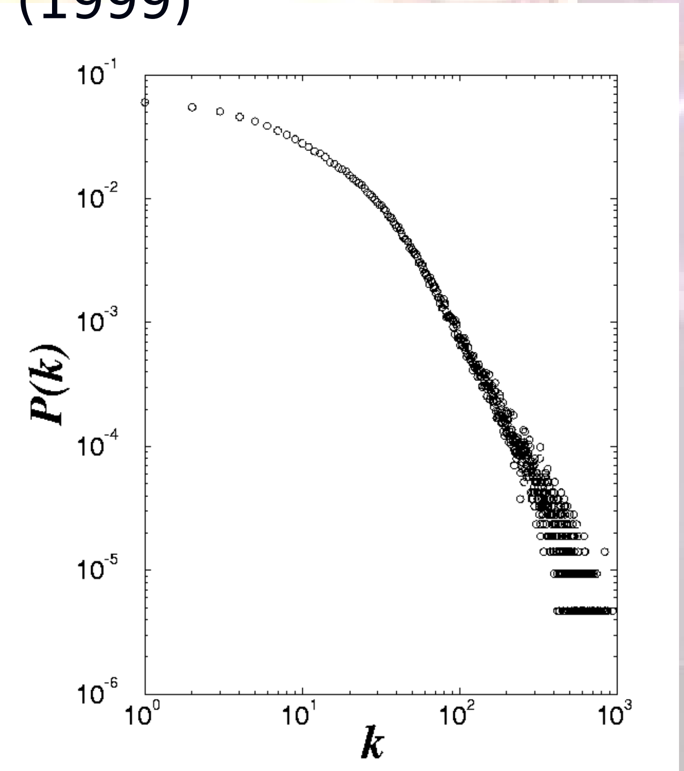


Days of Thunder (1990) Far and Away (1992) Eyes Wide Shut (1999)

N = 212,250 actors
⟨k⟩ = 28.78

$P(k) \sim k^{-\gamma}$

$\gamma = 2.3$



SCIENCE CITATION INDEX

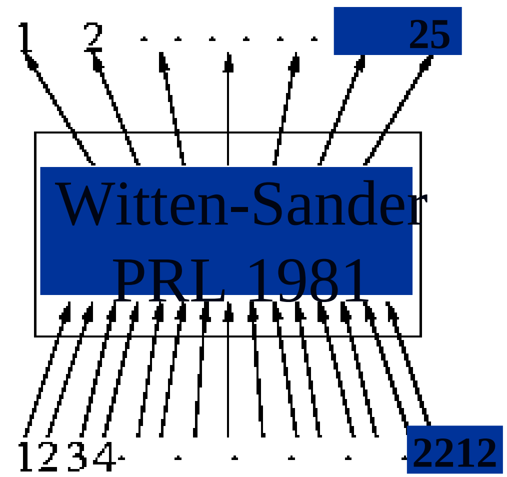
1,000 Most Cited Physicists, 1981-June 1997

Out of over 500,000 Examined

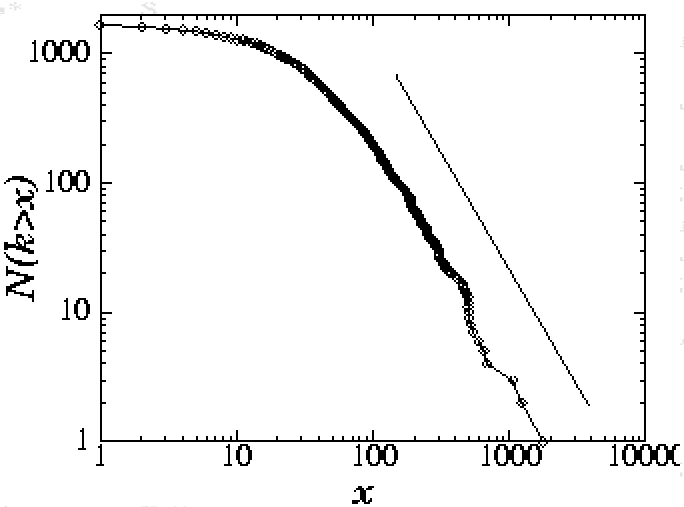
(see http://www.sst.nrel.gov)

Nodes: papers

Links: citations



1736 PRL papers (1988)



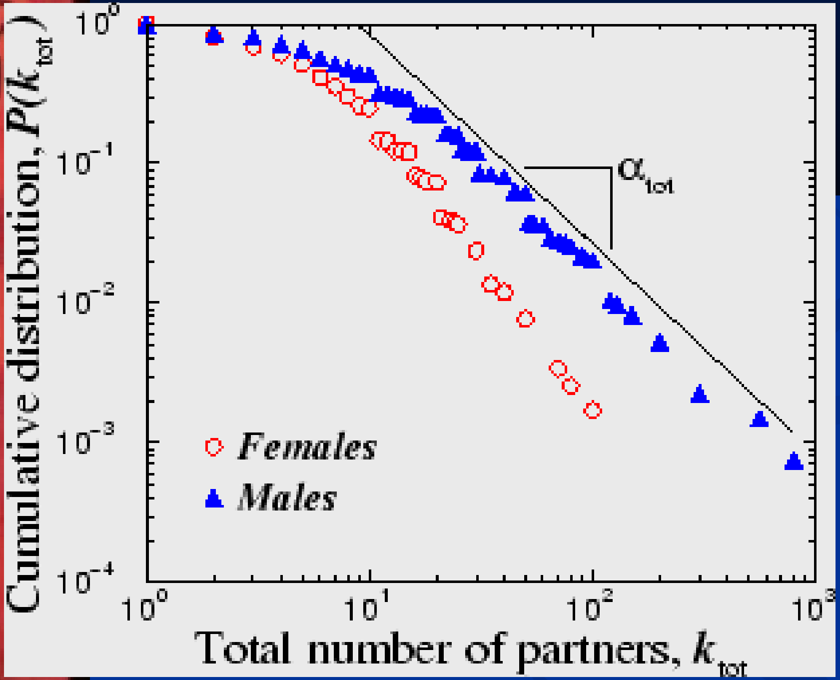
$$P(k) \sim k^{-\gamma}$$

$$(\gamma = 3)$$

(S. Redner, 1998)

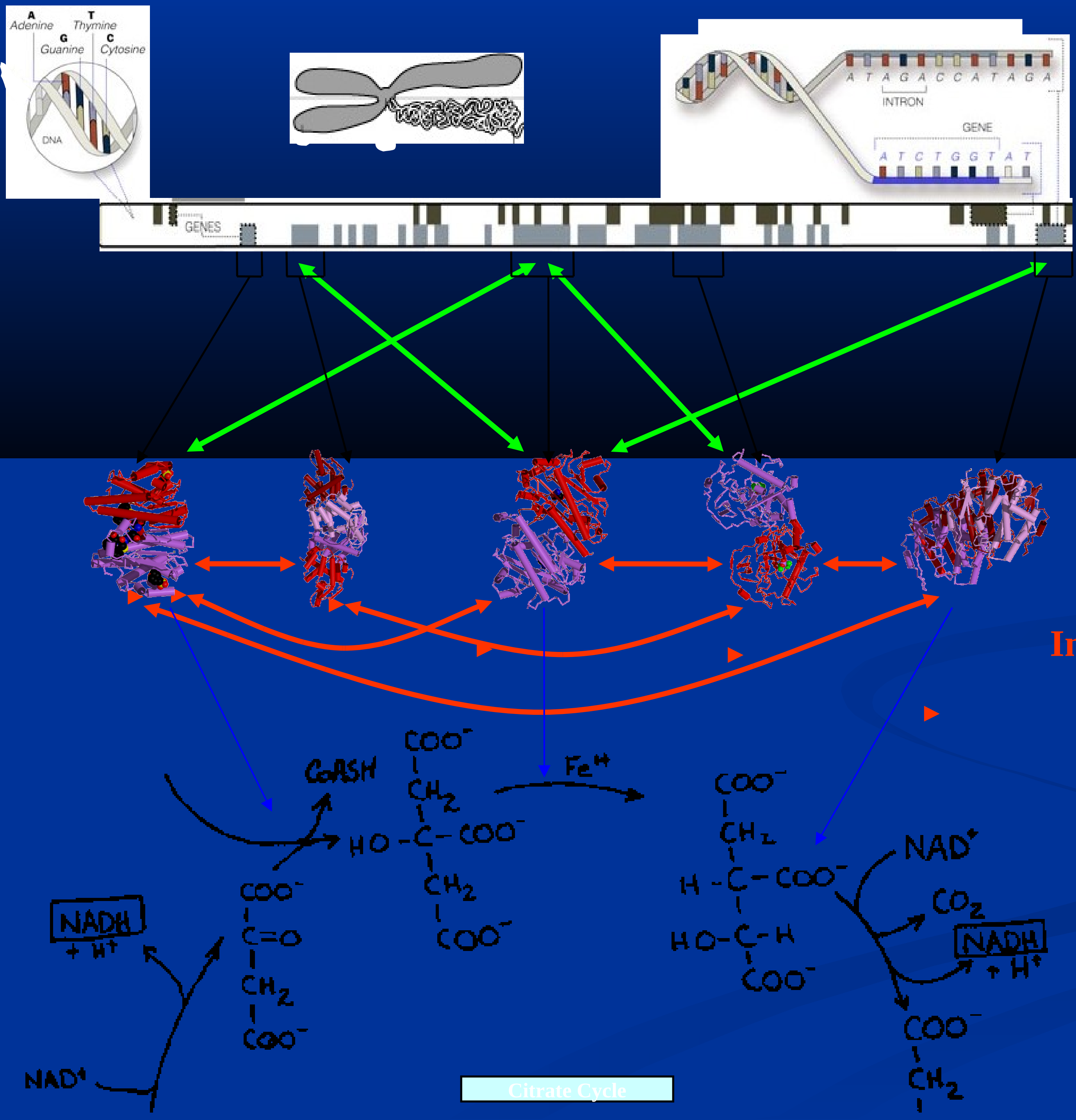
Sex-web

Nodes: people (Females; Males)
Links: sexual relationships



4781 Swedes; 18-74;
59% response rate.

Liljeros et al. Nature 2001



GENOMA

Interação proteina-gen

PROTEOMA

Interação proteina-proteina

enzimas

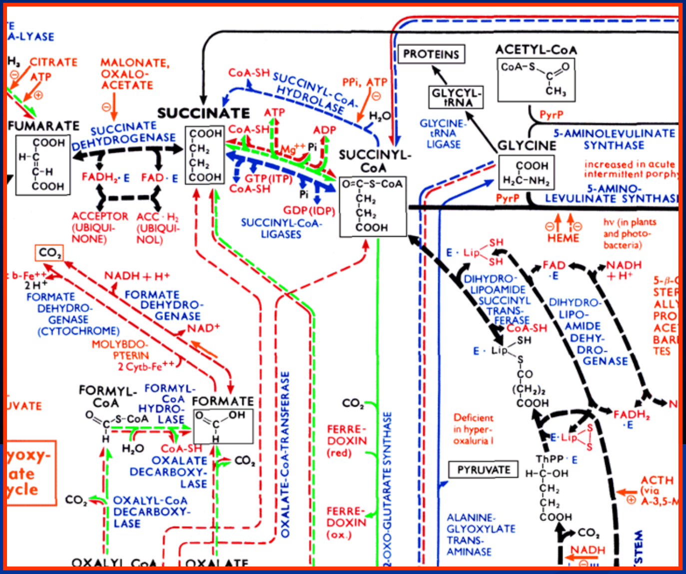
METABOLOMA

Reações bioquímicas

Cadeias de reações metabólicas (em verdade, **redes**)

# Reação	Fórmula	Enzima
1	2 ATP + Ac → AcCoA	acs
2	AcP → ATP + Ac	ackA
3	AcP → ATP + Ac	ackE
4	H_{ex} → ATP	atp_e
N	41.257 ATP + 3.547 NAD + 0.205 G6P … → Biomassa	-

Cadeias de reações metabólicas (em verdade, **redes**)



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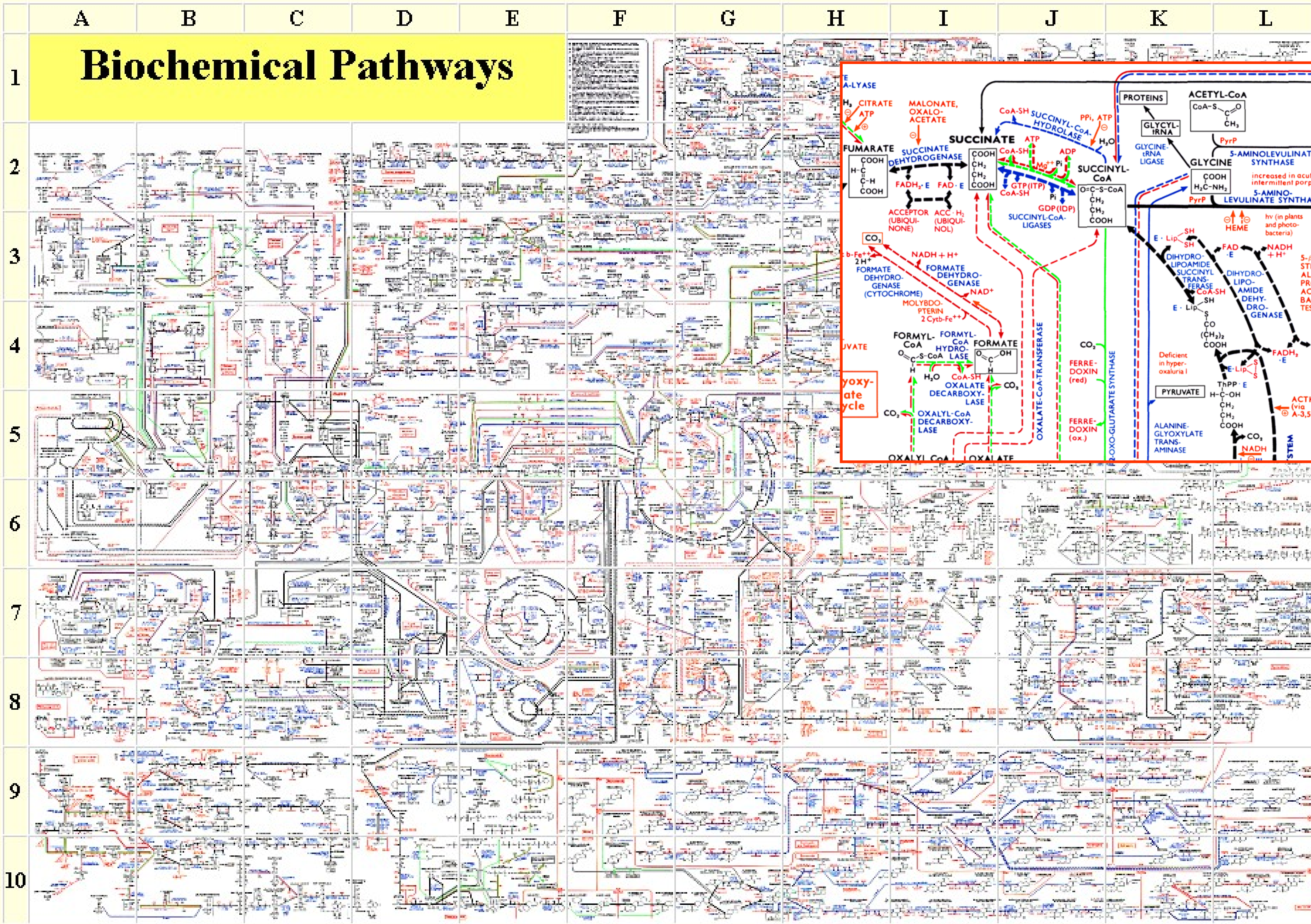
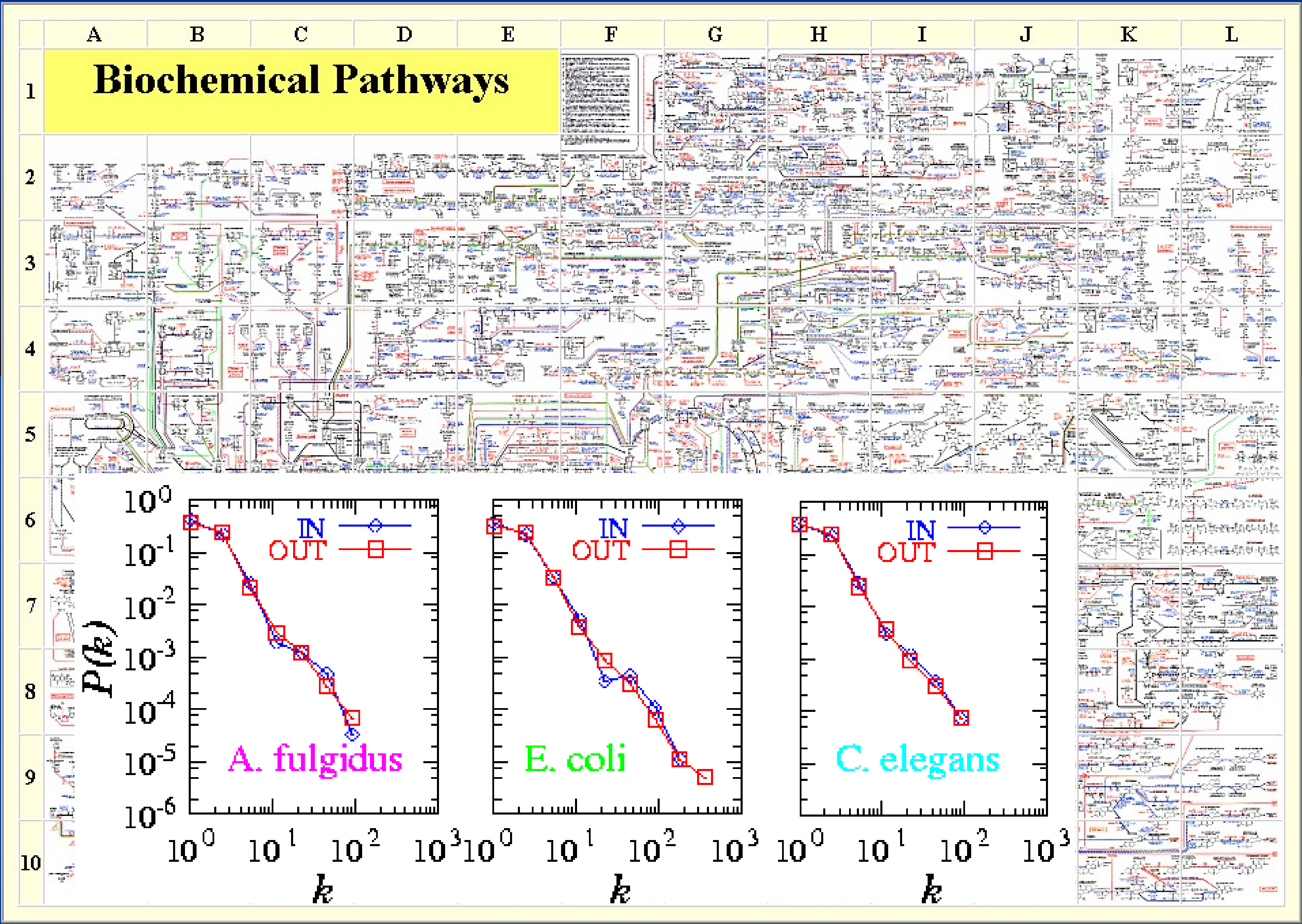
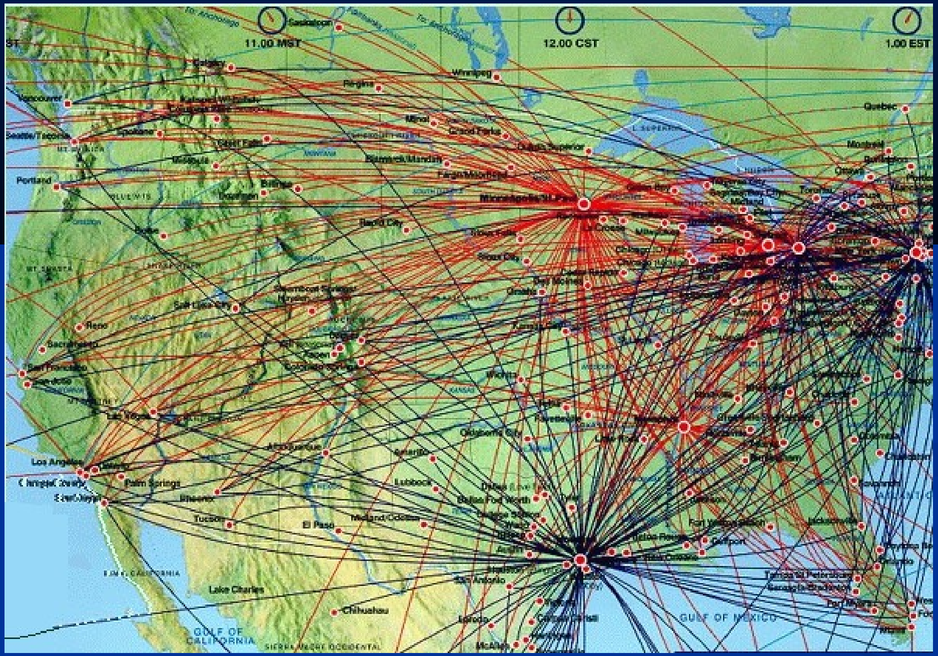
Biochemical Pathways



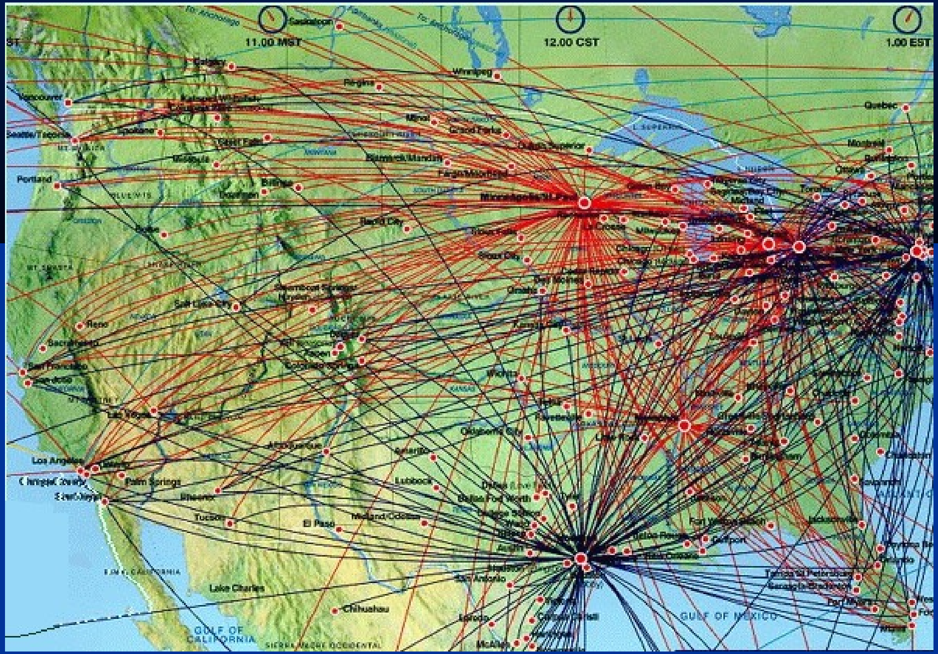

Biochemical Pathways
A
B
C
D
E
F
G
H
I
J
K
L
1
2
3
4
5
6
7
8
9
10
P(k)
k
IN
OUT
A. fulgidus
E. coli
C. elegans
10^0
10^-1
10^-2
10^-3
10^-4
10^-5
10^-6
10^0
10^1
10^2
10^3

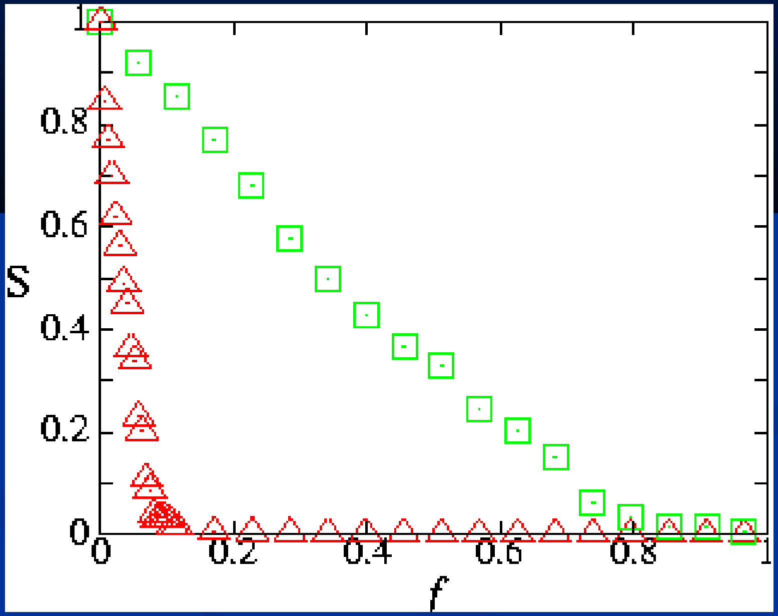
Vantagens e desvantagens da topologia scale-free



- Resistente ao malfuncionamento ocasional de seus elementos (sítios)
- Extremamente sensível a ataques intencionais dos Hubs.

Vantagens e desvantagens da topologia scale-free





- Resistente a uma falha ocasional de algum de seus elementos
- Extremamente sensível a ataques intencionais dos Hubs.

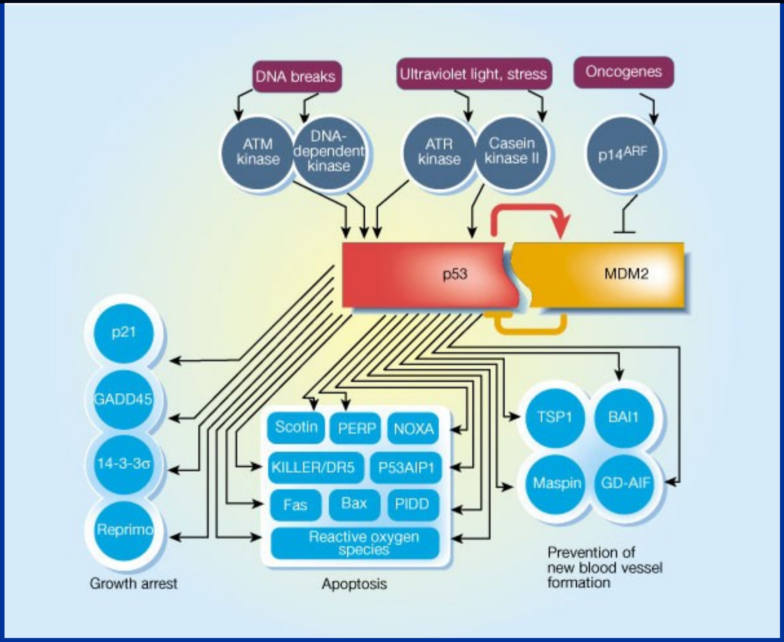
Nature **408** 307 (2000)

news and views feature

Surfing the p53 network

Bert Vogelstein, David Lane and Arnold J. Levine

The p53 tumour-suppressor gene integrates numerous signals that control cell life and death. As when a highly connected node in the Internet breaks down, the disruption of p53 has severe consequences.



…

"One way to understand the p53 network is to compare it to the Internet. The cell, like the Internet, appears to be a '**scale-free network**'."

Perguntas (V ou F, justifique)

1 ( ) Ao tratar sistemas com elementos interagentes como redes não perdemos informação

2 ( ) Os sítios de redes aleatórias possuem conectividades bastante distintas

3 ( ) Aeroportos são exemplos de topologias livre de escala

4 ( ) A distância típica entre dois sítios em uma rede aleatória com N sítios é proporcional a N